Evergreen
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Chapter-4

Laws of Motion

Ques- Explain the application of Inertia of Rest, Motion (horse cart problem , bus problem, carpet stick Horse riding)

→ Inertia of rest :- It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest & cannot start moving on its own. A body at rest opposes the force that tries to move it.
for eg:- suppose we are standing in a stationary bus & the driver starts the bus suddenly we get thrown backwards with a jerk.

⇒ Inertia of motion :- It is the inability to change by itself, its state of uniform motion i.e. a body in uniform motion can neither accelerate nor retard on its own & come to rest. A body in uniform motion opposes the force that tries to stop it.
for eg:- when a horse at full gallop stops suddenly, the rider falls forward on account of inertia of motion.

$P_2 - P_1 = mv - mv$

$= \Delta P$ (Change in momentum)

Time interval $= \Delta t$

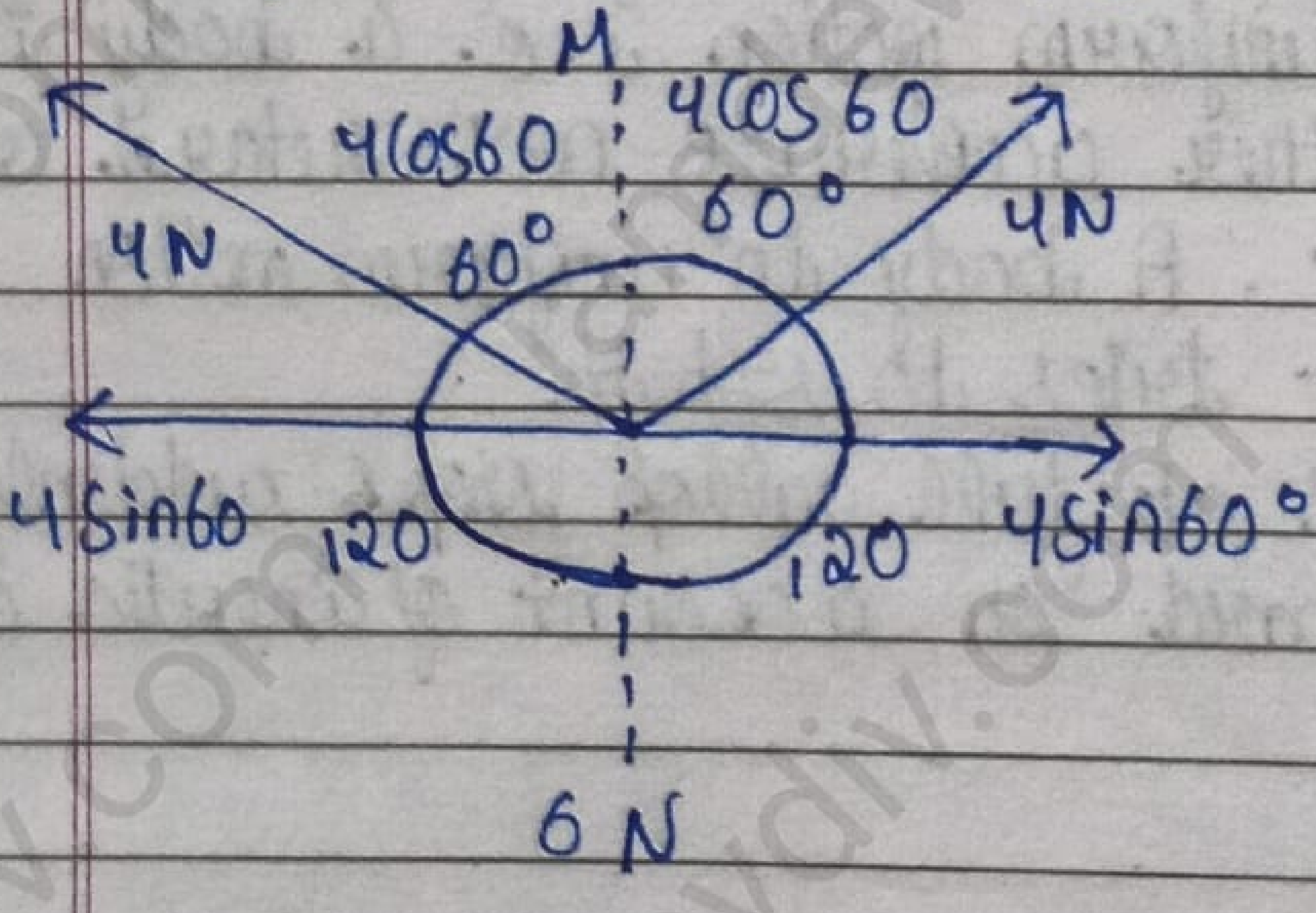
$$\frac{\Delta P}{\Delta t} = F$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Inertia :- Impulse $= F \times t$

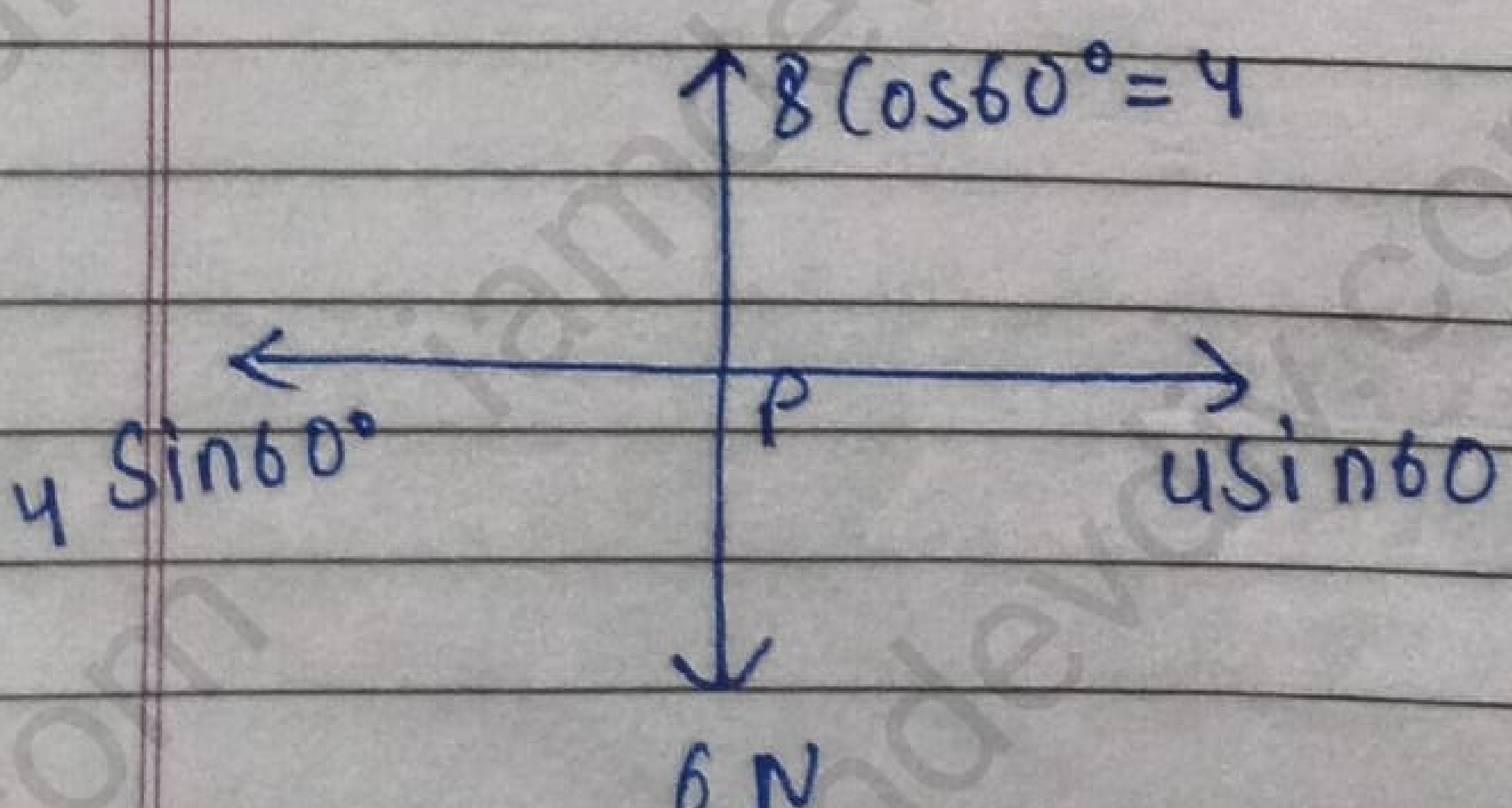
$$\downarrow F \propto \frac{1}{t} \uparrow$$

$F_{AB} = -F_{BA} \Rightarrow |\vec{F}_{AB}| = |\vec{F}_{BA}|$



$F \neq 0 \quad a = ✓$

$F = 0 \quad a = X$



$4 \neq 6$

$F = 2$

Newton's 2nd law is a real law

② $\frac{dp}{dt} = F = ma$ ——— ① Inertia law

——— ③ Action – Reaction

Case I $F = ma$

External force $= F = 0$

$ma = 0$

$m \neq 0$

$a = 0$

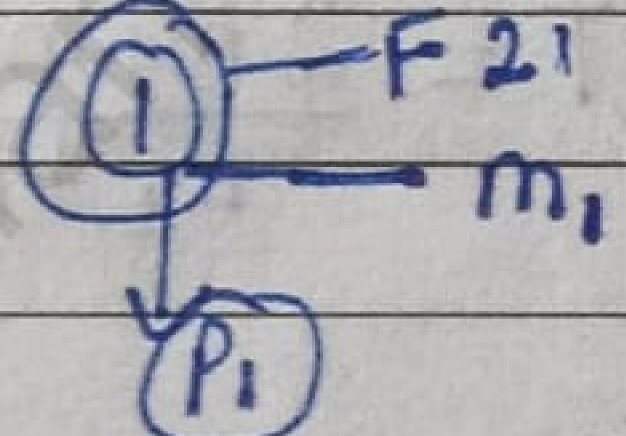
1st eq. of motion

$v = u + at$

$v = u + at$

$\boxed{v = u}$

Case II



Linear momentum

$P_N = P_1 + P_2$

$\frac{d}{dt} P_N = \frac{dP_1}{dt} + \frac{dP_2}{dt}$ — ①

from linear momentum conversion law

$\frac{dP_N}{dt} = 0$

$|\overline{F_{21}}| = |\overline{F_{12}}|$

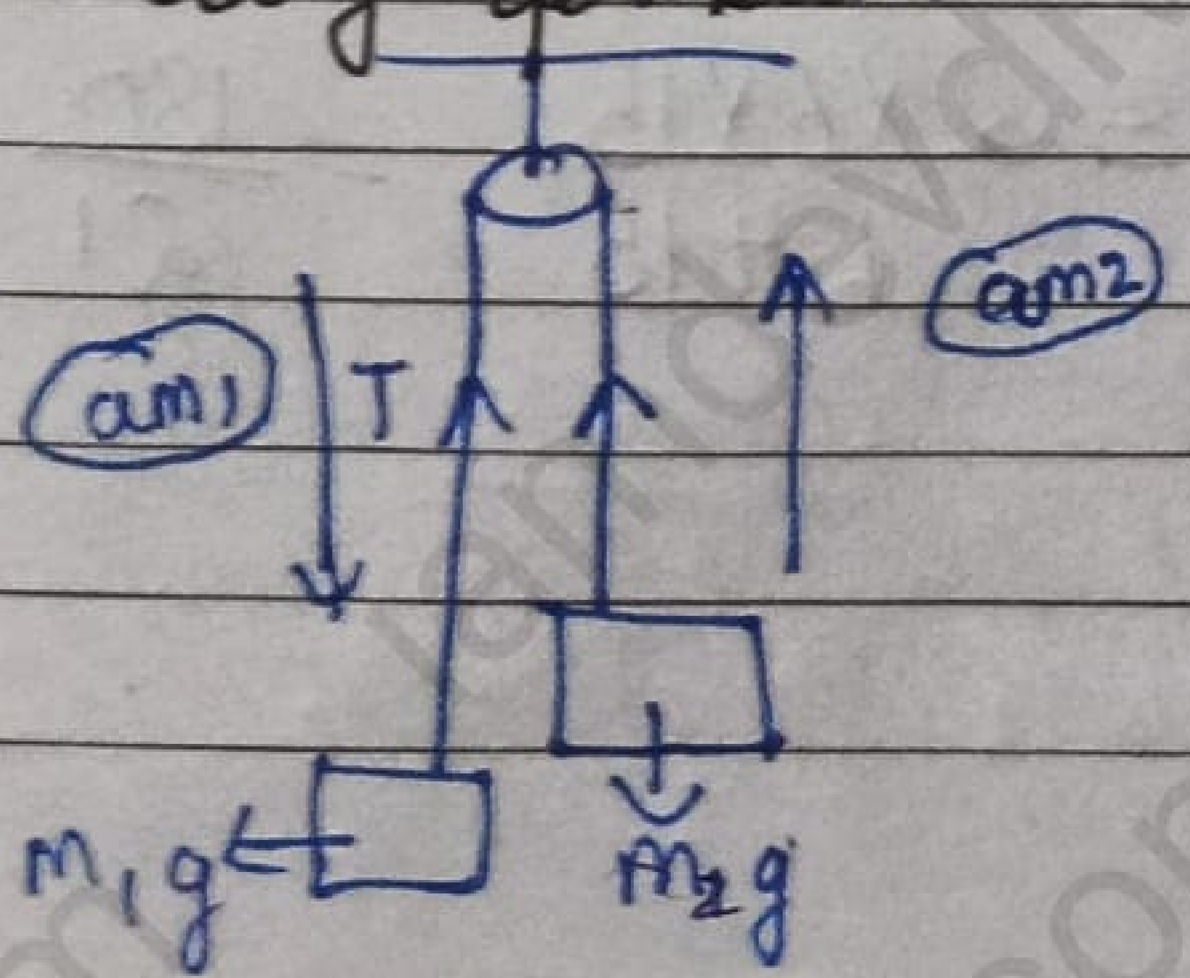
$\frac{dP_1}{dt} + \frac{dP_2}{dt} = 0$

$\frac{dP_1}{dt} = -\frac{dP_2}{dt}$

$F_{21} = -F_{12}$

$|\overline{F_{21}}| = |\overline{F_{12}}|$

Pulley problem



$a = ?$

$T = ?$

$$m_1 g - \cancel{T} = m_1 a \quad - (2)$$
$$\cancel{T} - m_2 g = m_2 a \quad - (3)$$
$$(m_1 - m_2) g = (m_1 + m_2) a$$

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

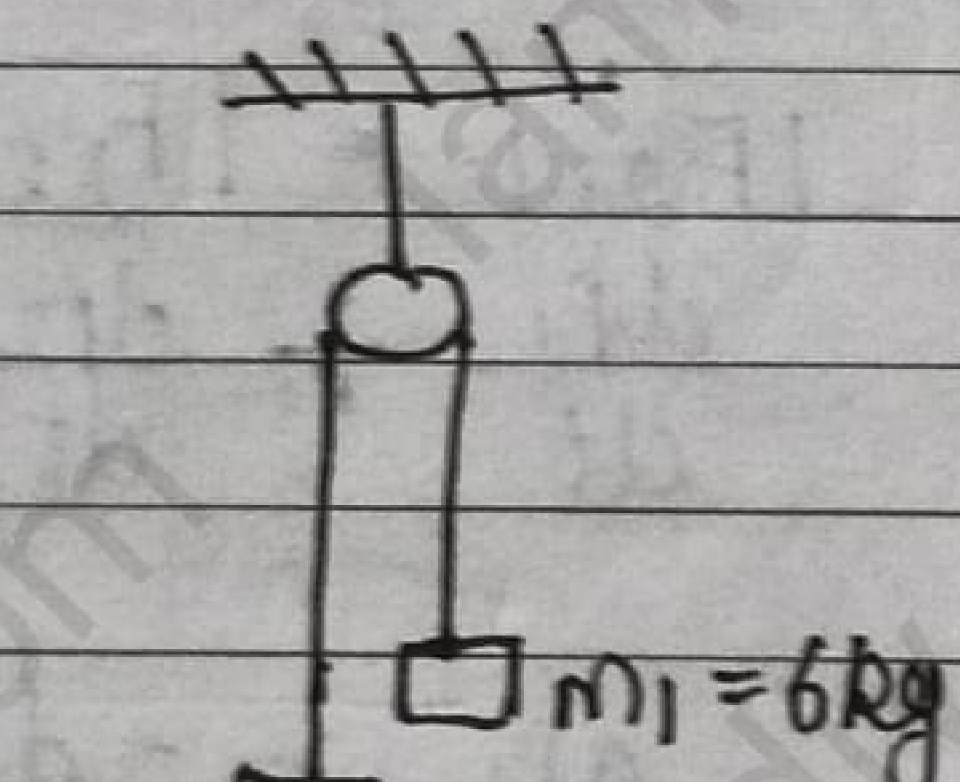
$$T = \frac{2 m_1 m_2}{m_1 + m_2} g$$

$$m_1 > m_2 ; \; a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$m_2 > m_1 , \; a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

$$T = \frac{2 m_1 m_2}{m_1 + m_2}$$

Ex:-



$T = ?$ $\quad \frac{600}{7}$

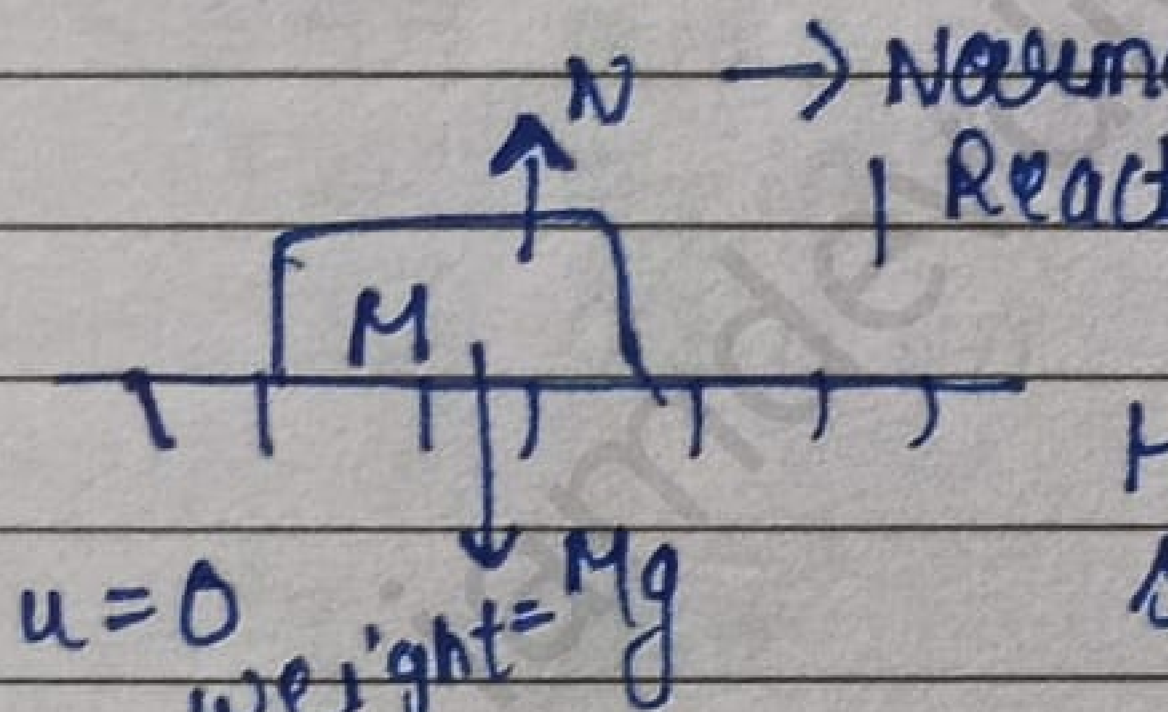
$a = ?$ $\quad \frac{30}{7}$

$$T = \frac{2 m_1 m_2}{m_1 + m_2} = \frac{2 \times 6 \times 15}{6 + 15} = \frac{180}{21} g$$

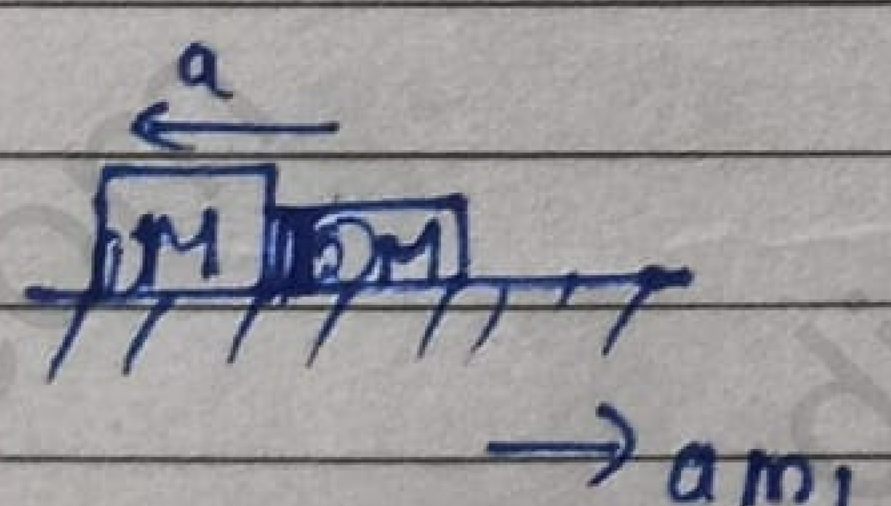
$\Rightarrow \frac{60}{7} g \Rightarrow \frac{600}{7}$

$a = \frac{m_2 - m_1}{21} = \frac{9}{21} g(10) = \frac{30}{7}$

Motion of bodies in contact



Ex



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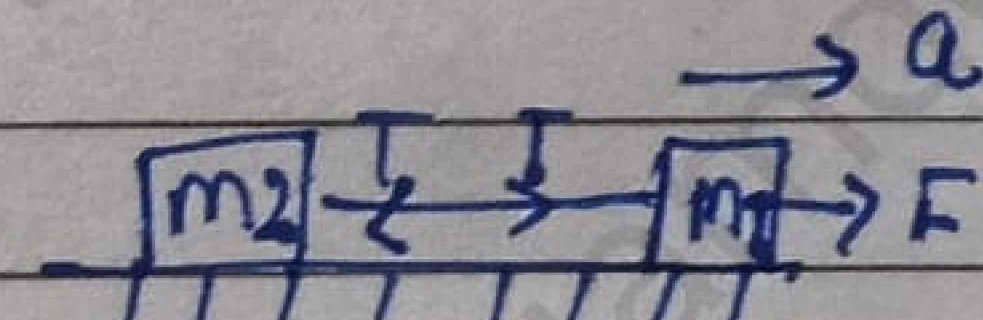


Case I

$T \leftarrow \boxed{m_1} \rightarrow F$

$F = T = m_1 a$ — ①

Case 2

$\uparrow 8N \rightarrow am_2$

$\boxed{m_2} \longrightarrow T$

$T = m_2 a$ — ②

$T = m_2 a$ — (2)

$F - T = m_1 a$ — (1)

from (1) & (2)

$F - m_2 a = m_1 a$

$F = m_1 a + m_2 a$

$F = (m_1 + m_2) a$

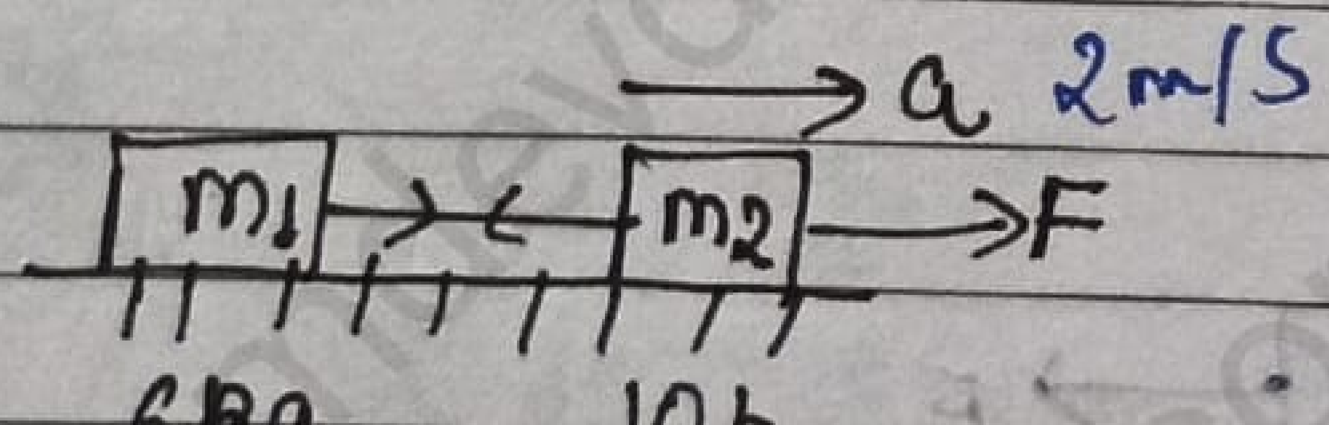
$\boxed{a = \dfrac{F}{m_1 + m_2}}$ — (3)

from (2) & (3)

$T = m_2 a$

$\boxed{T = \dfrac{m_2 F}{m_1 + m_2}}$

Problem I



$T = ?$

$F = ?$

Equation = ?

$F - T = m_2 a$

$\boxed{F - T = 10a}$ — (1)

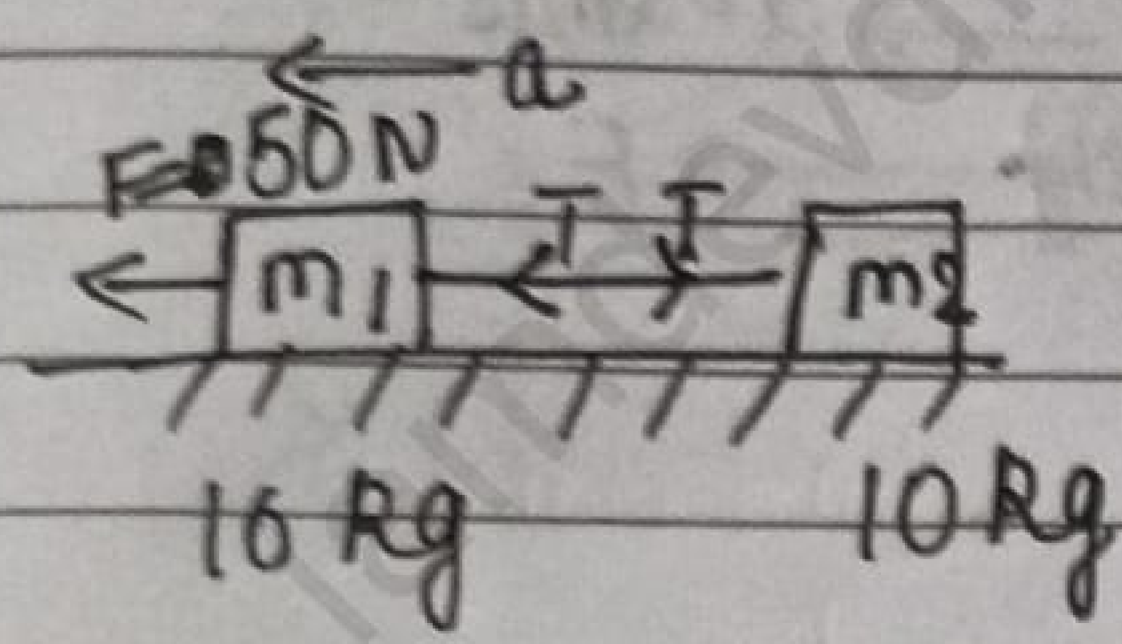
$T = m_1 a$

$\boxed{T = 6a}$ — (2)

$F - 10a = 6a \Rightarrow 6a + 10a \Rightarrow 16a \Rightarrow T = 16 \times 2 = 32$

$\boxed{F = 32M}$

$T = m_1 a \Rightarrow T = 6 \times 2 = 12$



$a = ?$

$T = ?$

$F = 50N$

$m_1 = 16kg$

$m_2 = 10\ kg$

$F - T = m_2 a$

$50 - T = 10$

$T = 50 - 10$

$T = 40$

$T = m_1 a$

$40 = 16a$

$\frac{40}{16} = a$

$a = \frac{F}{m_1 + m_2}$

$= \frac{50}{16+10}$

$= \frac{\cancel{50}}{\cancel{26}}$

$= \frac{25}{13}$

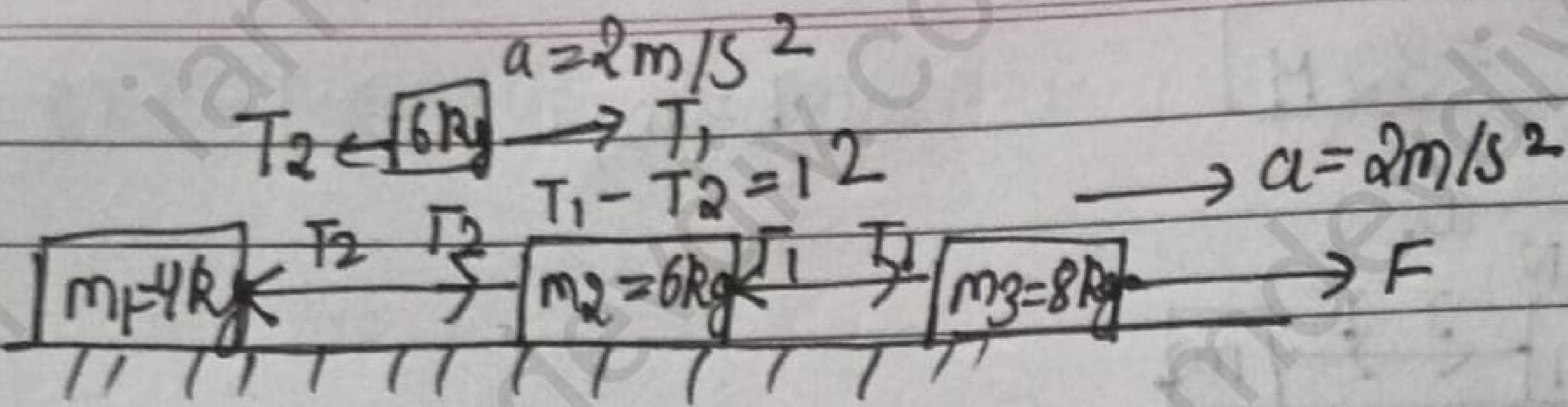
$T = \frac{m_2 f}{m_1 + m_2}$

$= \frac{10 \times 50}{16+10}$

$= \frac{\cancel{500}}{\cancel{26}}$

$= \frac{250}{13}$

#



$a = 2 m/s^2$

$4kg = T_2$

$T_2 = 8$

$F - T_1 = 16$ — (1)

$T_1 - T_2 = 12$ — (2)

$T_2 = 8$ — (3)

$T_1 - 8 = 12$

$T_1 = 12 + 8$

$T_1 = 20$

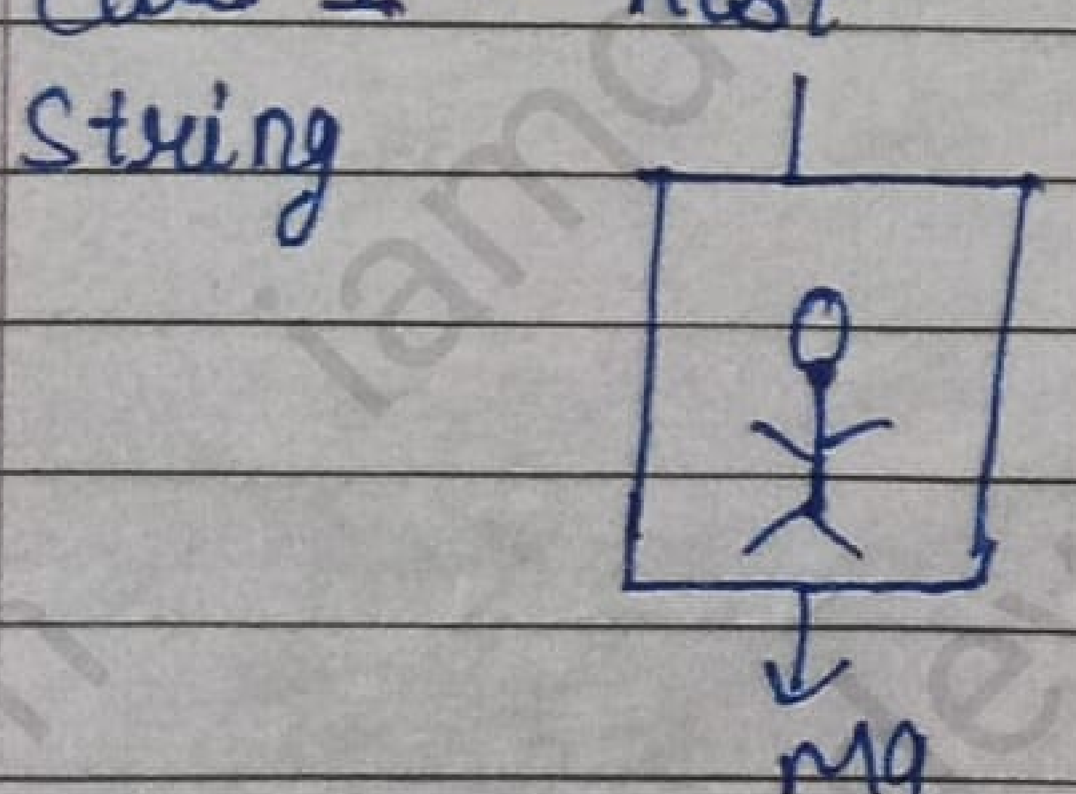
$F - 20 = 16$

$F = 16 + 20$

$= 36$

★ Effective ~~or~~ apparent weight of a man in a lift

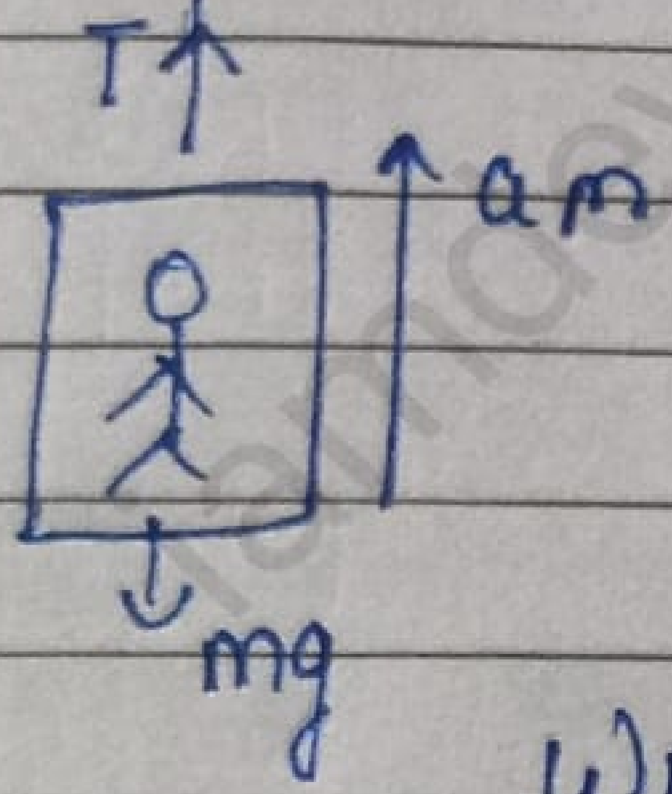
Case I - Rest



$u = 0$

$\boxed{W_{App} = W_{actual}}$

Case 2 Accelerated ↑ (upward)



$W_{APP} \neq W_{Act}$

$W_{APP} > W_{Acc}$.

$W \uparrow$

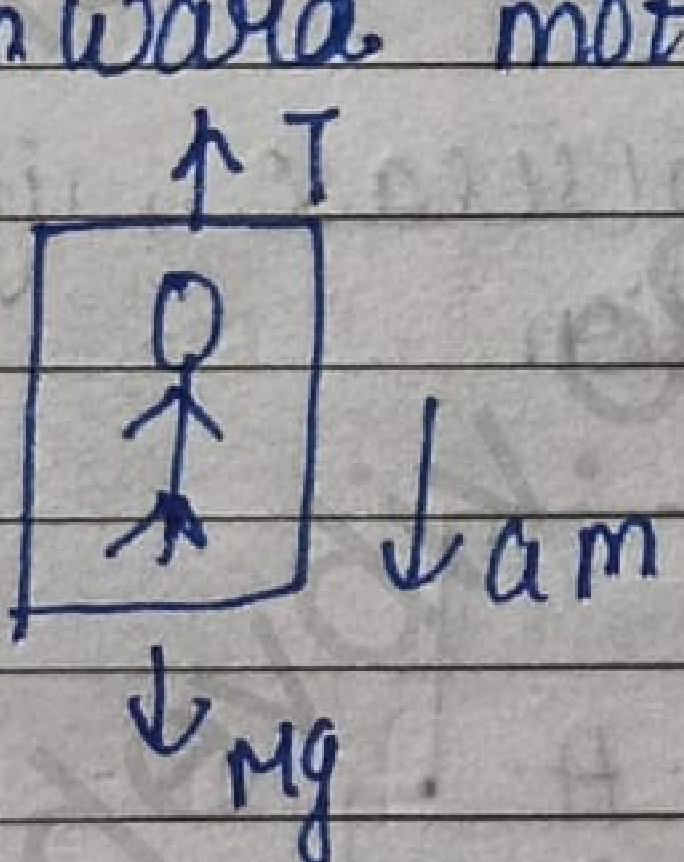
$\cancel{T - Mg = Ma}$ $\quad T - Mg = Ma$

$T = Mg + Ma$

$T = m(g+a)$

$$\boxed{W_{APP} = m(g+a)}$$

Case 3 downward motion ↓



$W_{APP} < W_{ACC}$

$W \downarrow$

$mg - T = ma$

$T = mg - ma$

$$\boxed{W_{APP} = m(g-a)}$$

Case 4 → Lift falling freely

$$W_{APP} = m(g-a) \downarrow$$

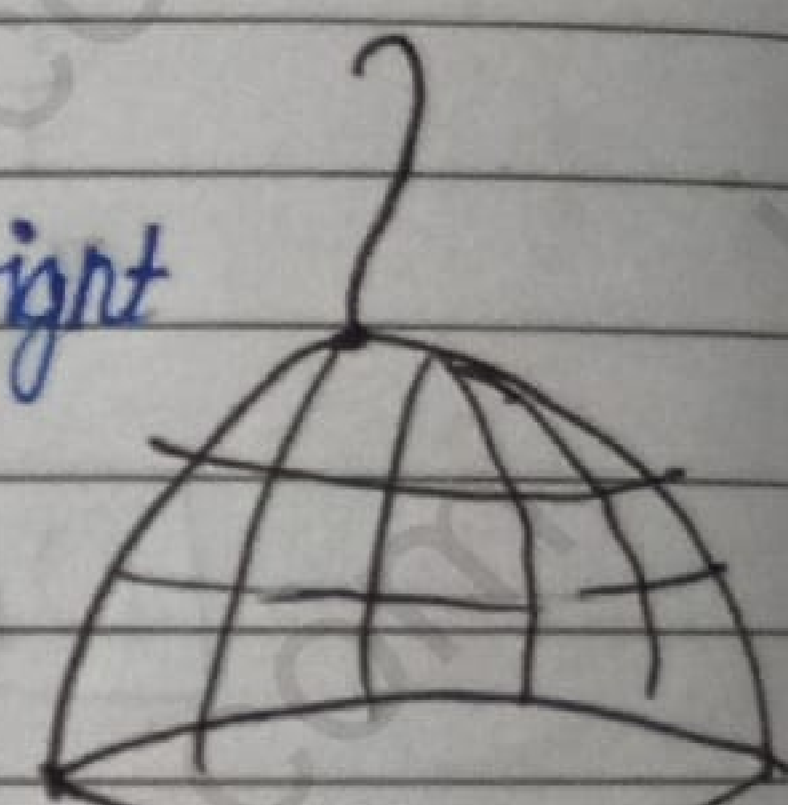
$$a = g$$

$$W_{APP} = m(g-g)$$

$$\boxed{W_{APP} = 0}$$

☆ Imp

Bird - Cage Problem



A bird is sitting on the base in an air tight cage. Now if the bird start flying then

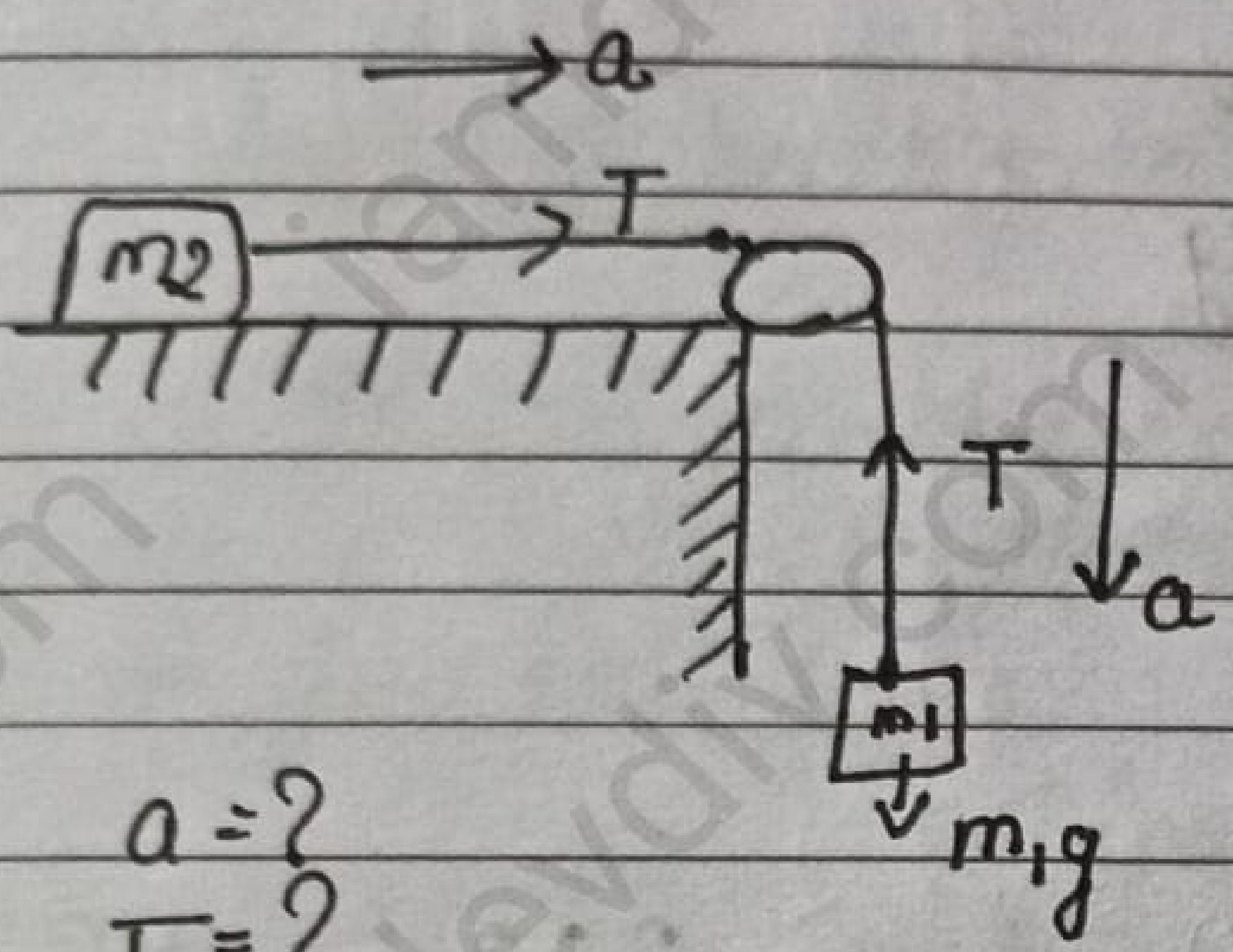
1. Weight of the system will not change, if bird flying with the <u>constant velocity</u>.
2. Weight of the system will increase, if the bird flying with upward acceleration

$$\boxed{\begin{array}{l} \text{Same} \rightarrow C.V. \\ \text{Increase} \rightarrow U.A. \\ \text{Decrease} \rightarrow D.A \end{array}}$$

<u>Note:</u> A bird is sitting on the base in a wire cave. Now, If the bird flying upward its weight will decrease in all the cases.

☆ Special Cases

Case I



$a = ?$
$T = ?$

$$m_1g - T = m_1a \quad -(1)$$

$$\boxed{T = m_2a} \quad -(2)$$

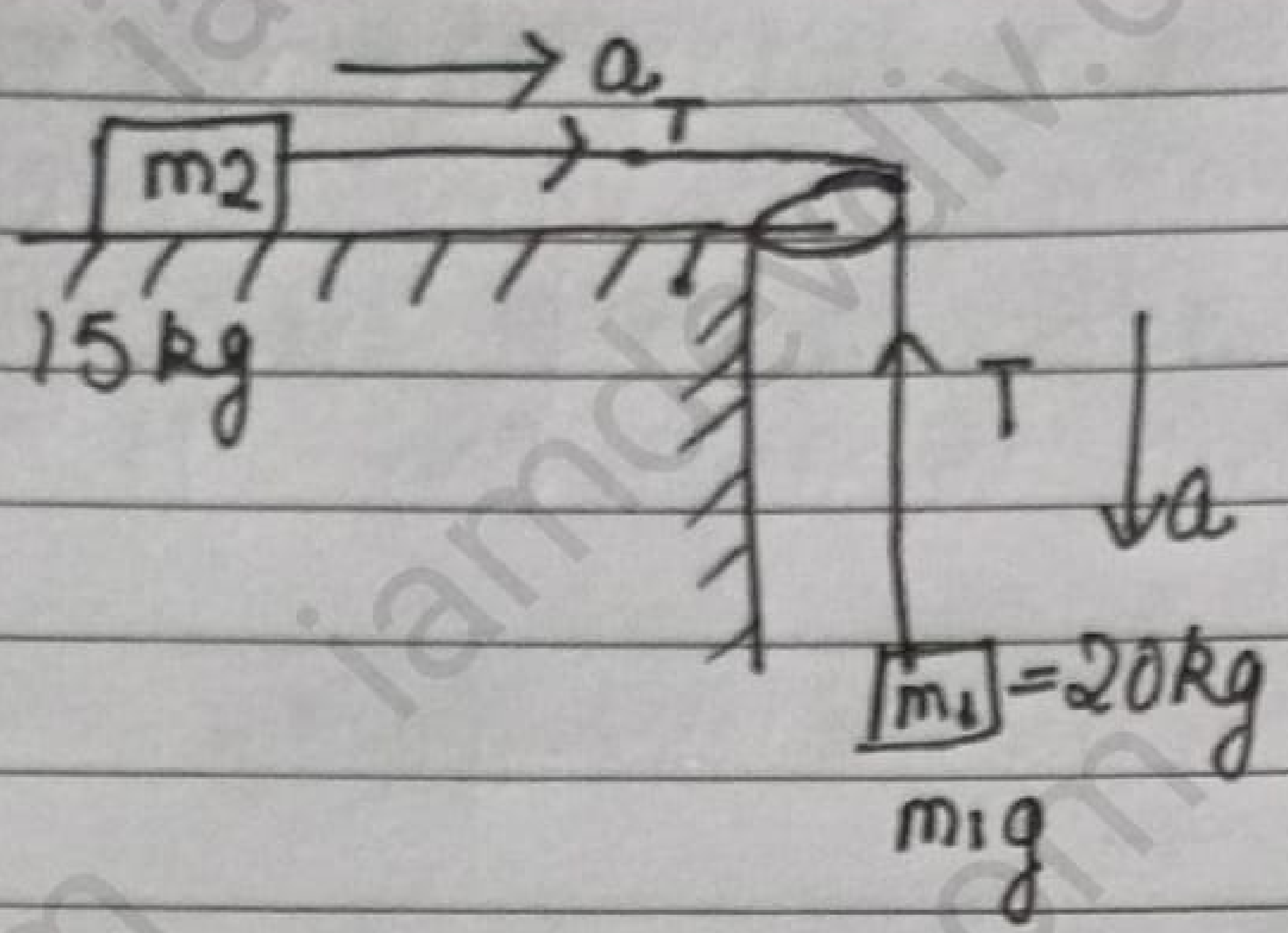
from (2) & (1)

$$m_1g - m_2a = m_1a$$
$$m_1g = m_1a + m_2a$$
$$m_1g = (m_1 + m_2)a$$
$$\boxed{a = \frac{m_1g}{m_1+m_2}}$$

from again eq (2)

$$\boxed{T = \frac{m_1m_2}{m_1+m_2}g}$$

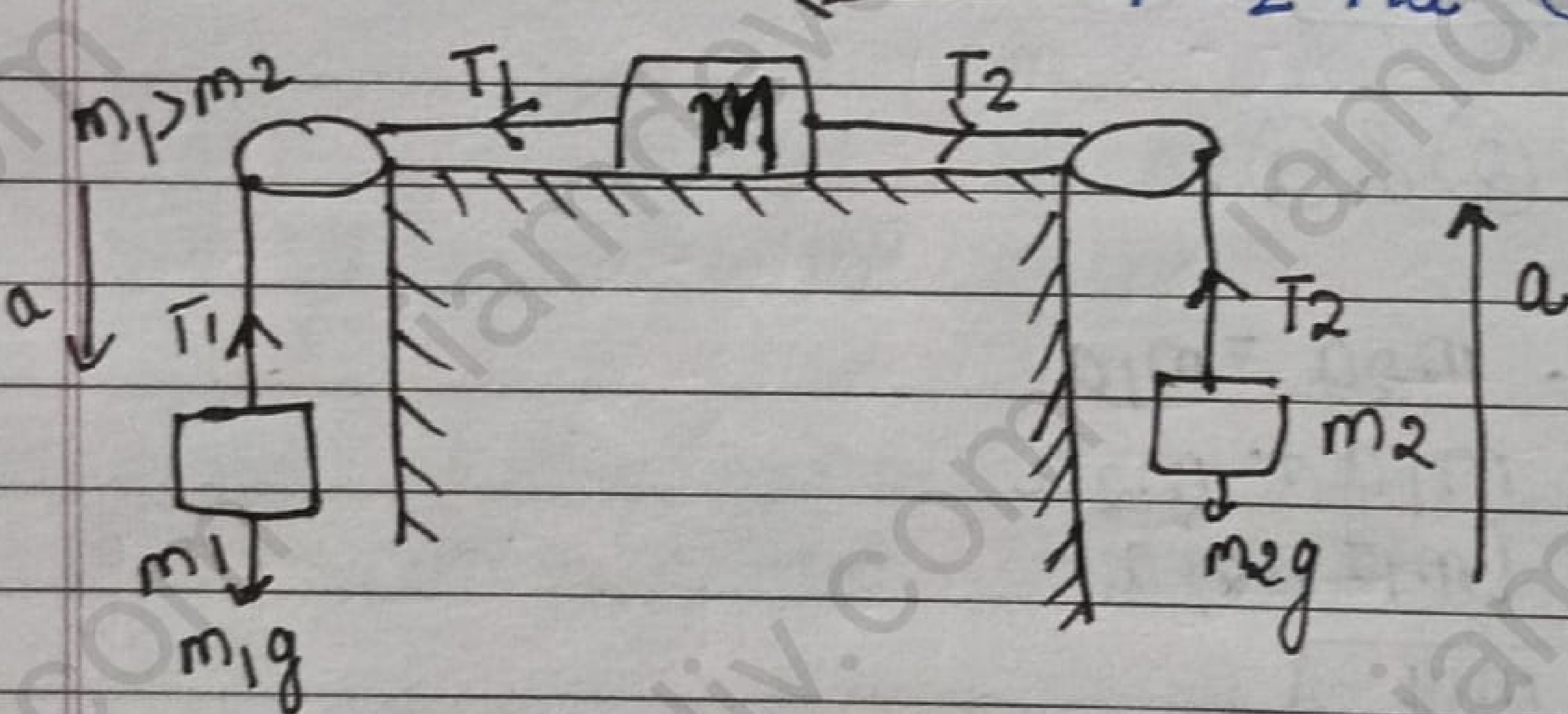
Ques -



$a = ?$ $\quad T = ?$

$$a = \frac{m_1 g}{m_1 + m_2} \Rightarrow \frac{20 \times 10}{20 + 15} = \frac{200}{35} = \frac{40}{7}$$

a $\leftarrow$ $T - T_2 = Ma$ - ③



$m_1g - T_1 = m_1a$ - ②

$T_2 - m_2g = m_2a$ - ①

$$\cancel{T_2} - m_2g = m_2a \quad - ①$$
$$m_1g - \cancel{T_1} = m_1a \quad - ②$$
$$\cancel{T_1} - \cancel{T_2} = Ma \quad - ③$$
$$\overline{m_1g - m_2g = (m_1 + m_2 + M)a}$$

$$g(m_1 - m_2) = (m_1 + m_2 + M)a$$

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2 + M)}$$

Friction

1. Define Friction force with its type

2. Define Angle of friction and angle of Repose.

3. Find a derivation for acceleration for a moving body on a inclined plane in downward / upward due to friction force

4. Explain ~~banking~~ ~~ending~~ banking of road

Ans1- The force that opposes the motion of a solid object over another is called frictional force.
There are mainly 3 type of friction

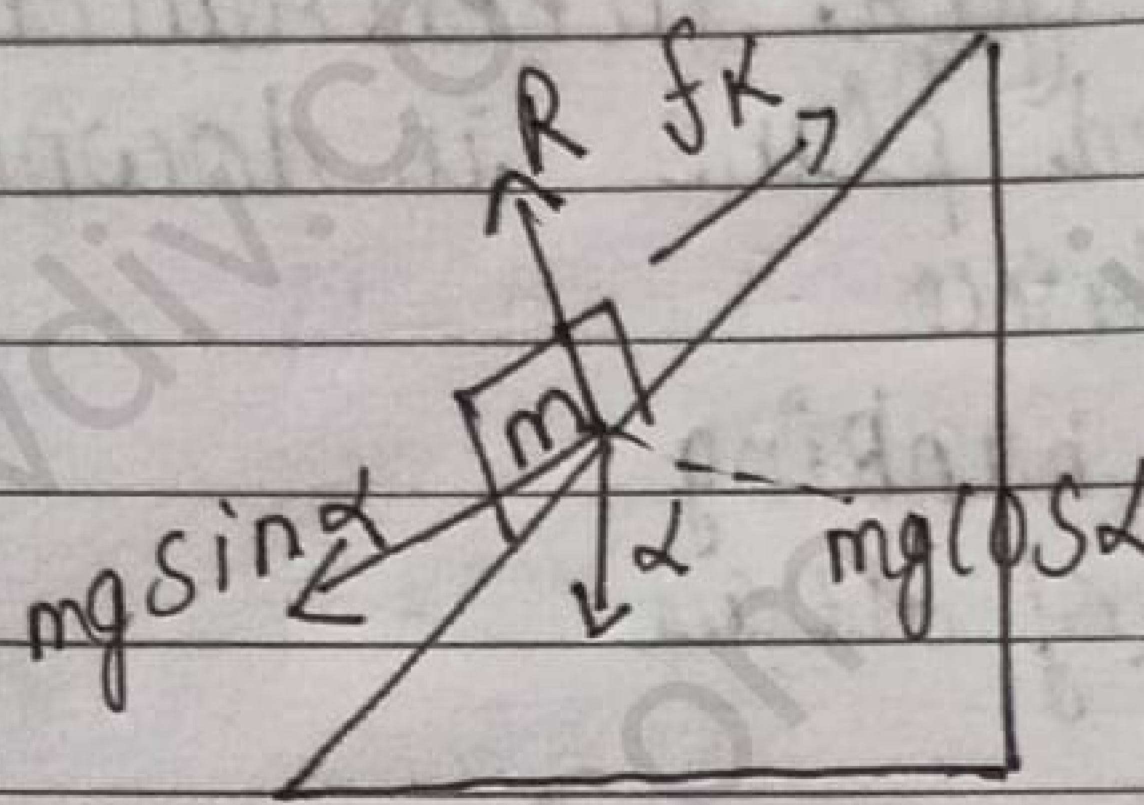
1. Static frictional force
2. Limiting frictional force
3. Kinetic frictional force.

Ans2- The angle between the normal reaction force and the resultant force of normal reaction force and friction when an object just begins to move is called angle of friction.

Angle of repose is defined as the minimum angle of an inclined plane which causes an object to slide down the plane is called angle of repose.

Ans 4 - The phenomenon in which the outer edges are raised for the curved roads above the inner edge to provide the necessary centripetal force to the vehicles so that they take a safe turn.

On a rough inclined plane



Case I

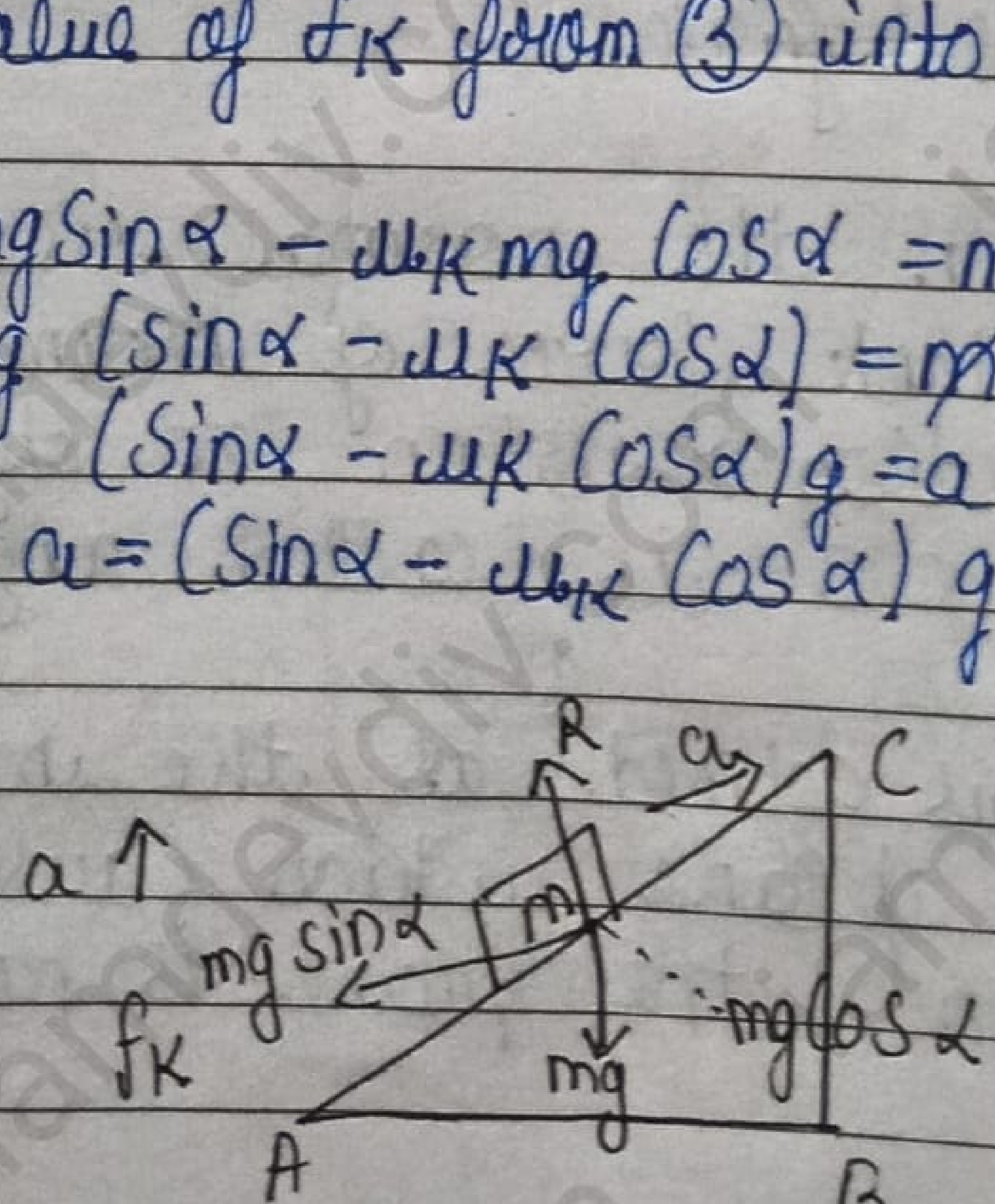
$$R = mg\cos\alpha \quad - (1)$$
$$mg\sin\alpha - f_K = ma \quad - (2)$$

$$f_K = \mu_K R$$
$$f_K = \mu_K mg\cos\alpha \quad — (3)$$

putting value of f_K from (3) into (2)

$$mg\sin\alpha - \mu_K mg\cos\alpha = ma$$
$$\cancel{m}g(\sin\alpha - \mu_K\cos\alpha) = \cancel{m}a$$
$$(\sin\alpha - \mu_K\cos\alpha)g = a$$
$$a = (\sin\alpha - \mu_K\cos\alpha)\,g$$

Case 2



$R = mg\cos\alpha$ - (1)

$mg\sin\alpha + f_K = ma$ - (2)

$f_K = \mu_K R$

$f_K = \mu_K mg\cos\alpha$ - (3)

putting value of f_K from (3) into (2)

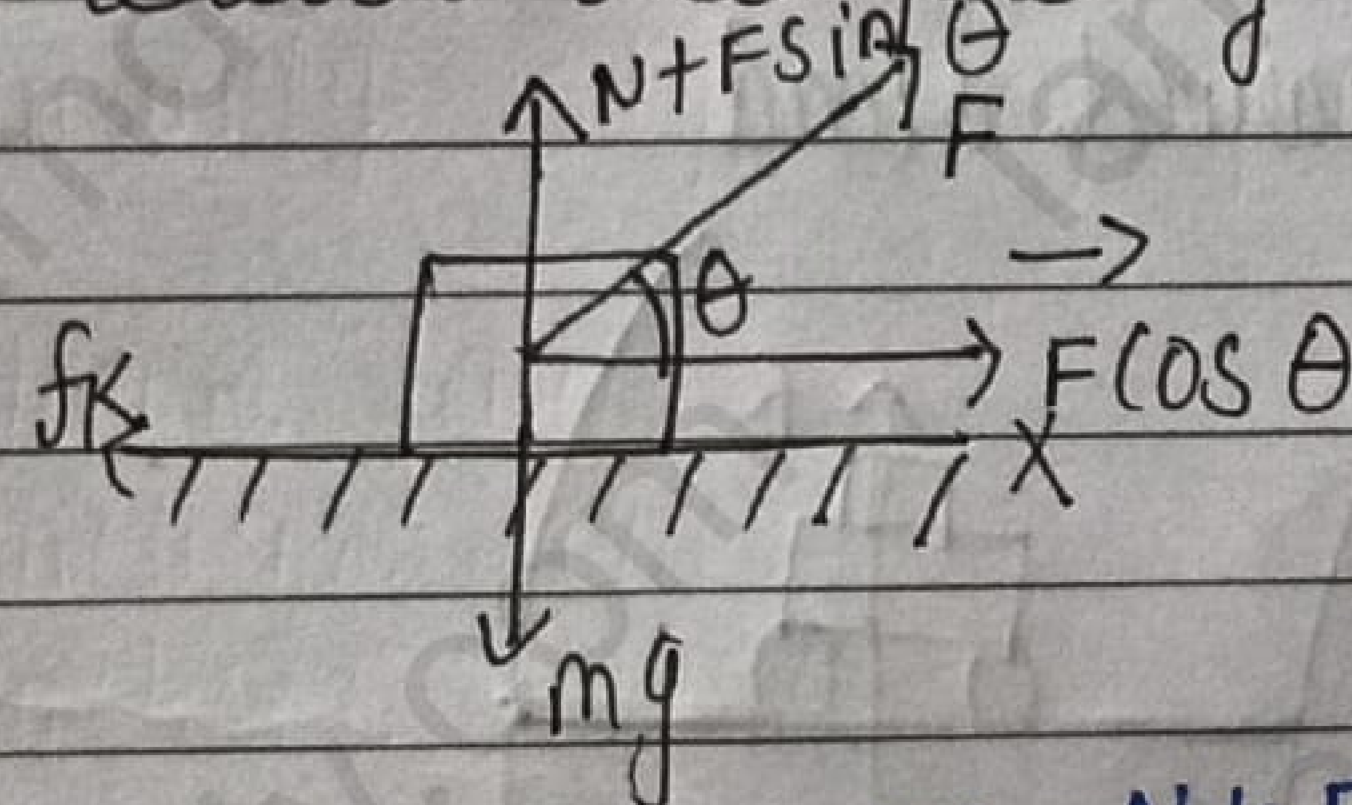
$mg\sin\alpha + \mu_K mg\cos\alpha = ma$

$\cancel{m}g(\sin\alpha + \mu_K\cos\alpha) = \cancel{m}a$

$(\sin\alpha + \mu_K\cos\alpha)g = a$

$$\boxed{a = (\sin\alpha + \mu_K\cos\alpha)g}$$

pulling is easier than pushing



$\omega = F\cos\theta$
$\theta = 0$

$mg = N + F\sin\theta$

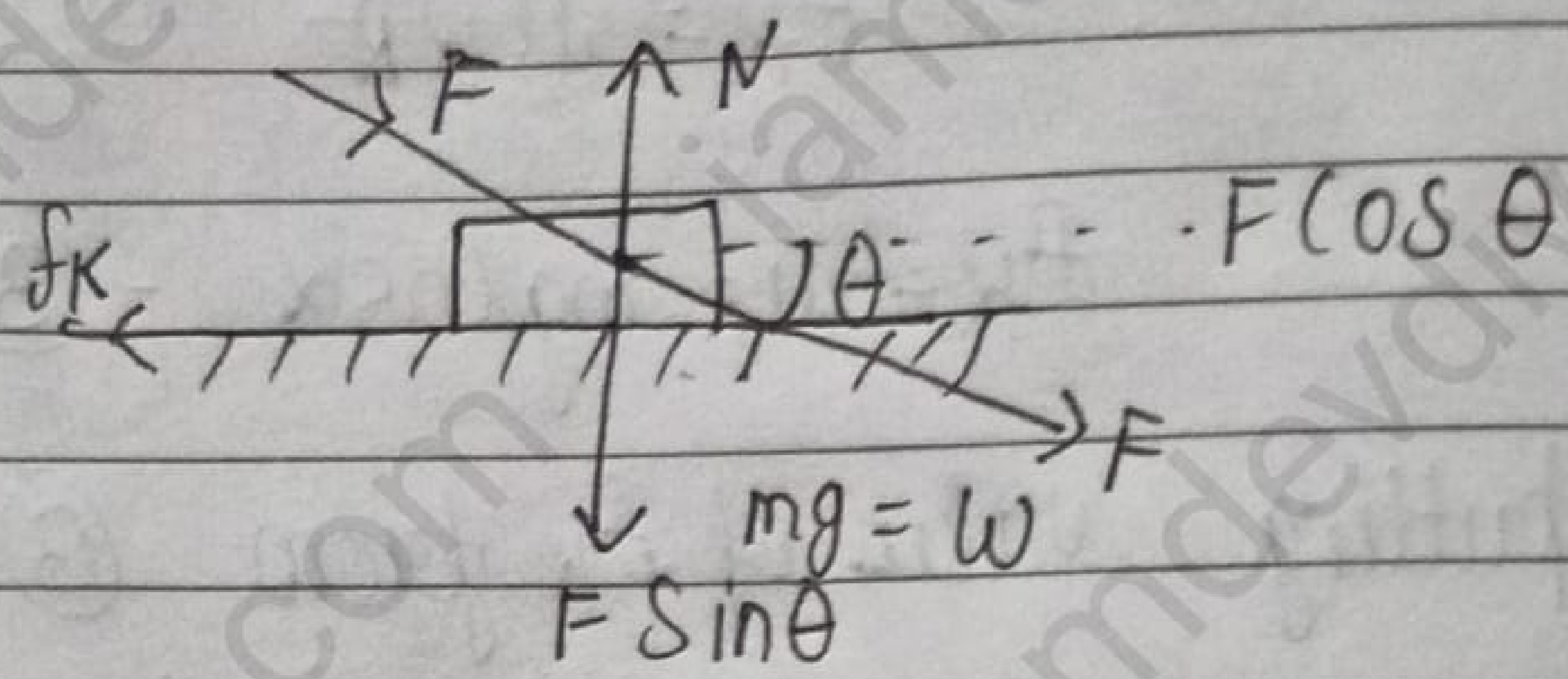
$$\boxed{N = mg - F\sin\theta}$$

$f_K = \mu_K N$

$f_K = \mu_K(mg - F\sin\theta)$

$$\boxed{f_K = \mu_K(\omega - F\sin\theta)}$$ - (1)

Case of Pushing



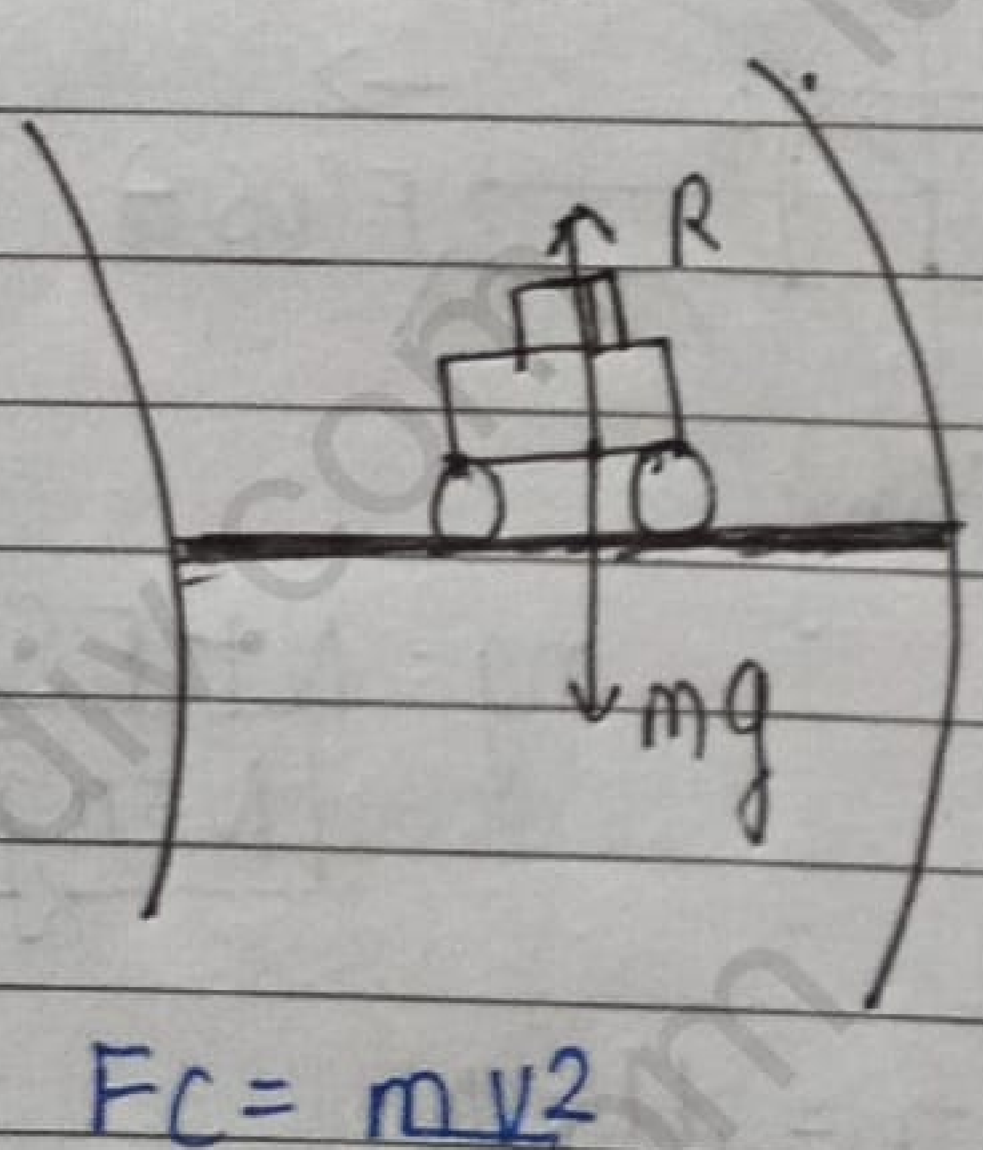
$N = mg + F\sin\theta$

$f_K' = \mu_K N$

$\boxed{f_K' = \mu_K (mg + F\sin\theta)}$ - (2)

$\boxed{f_K' > f_K}$

Rounded a level curved road



$F_C = \frac{mv^2}{r}$

$F_C = mr\omega^2$

$\omega = 2\pi n$

$F_C = 4\pi^2 n^2 m r$

$F_C < F_f$

$\frac{mv^2}{r} \leq uR$

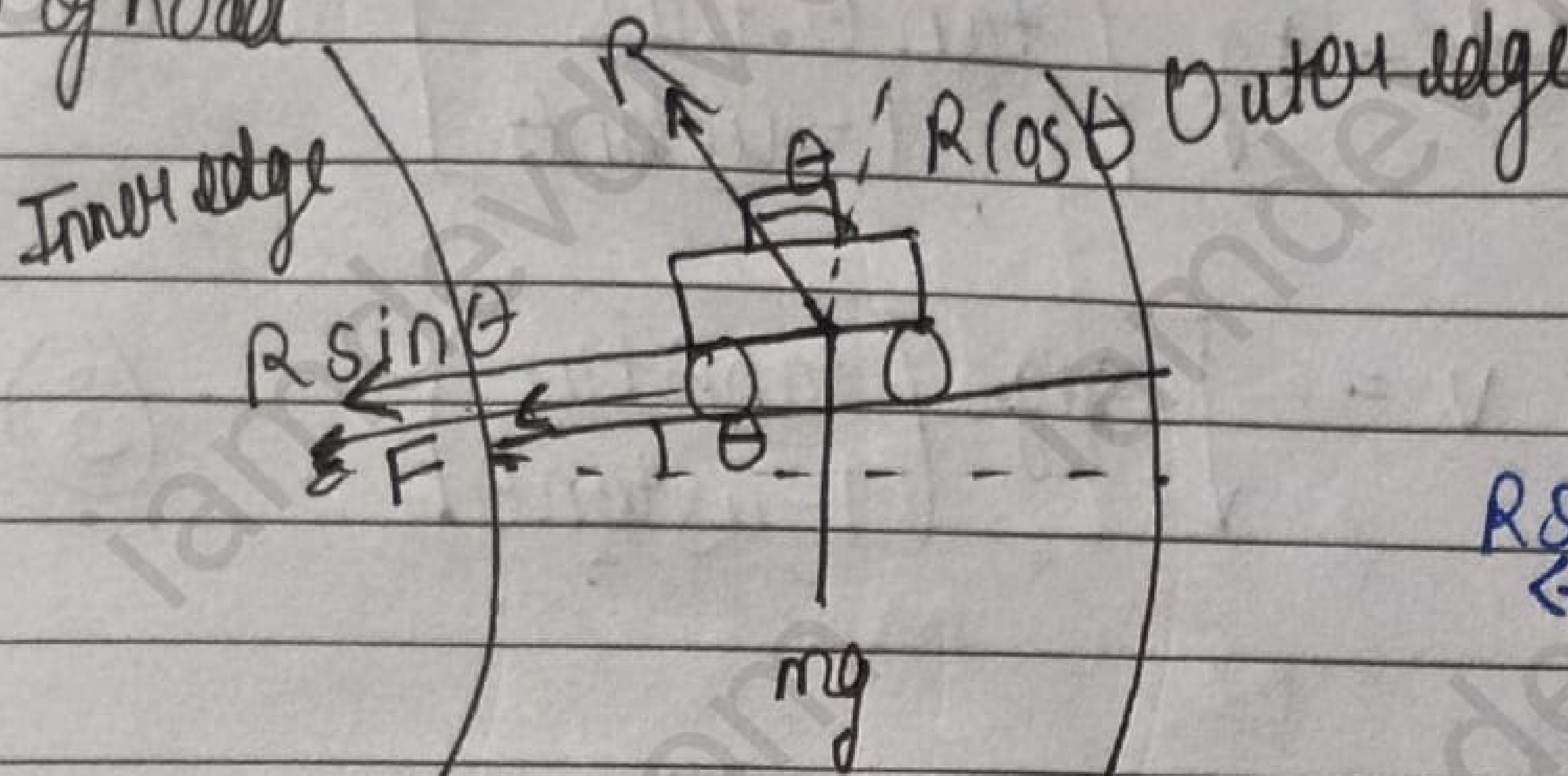
$\frac{\cancel{m}v^2}{r} \leq u\cancel{m}g$

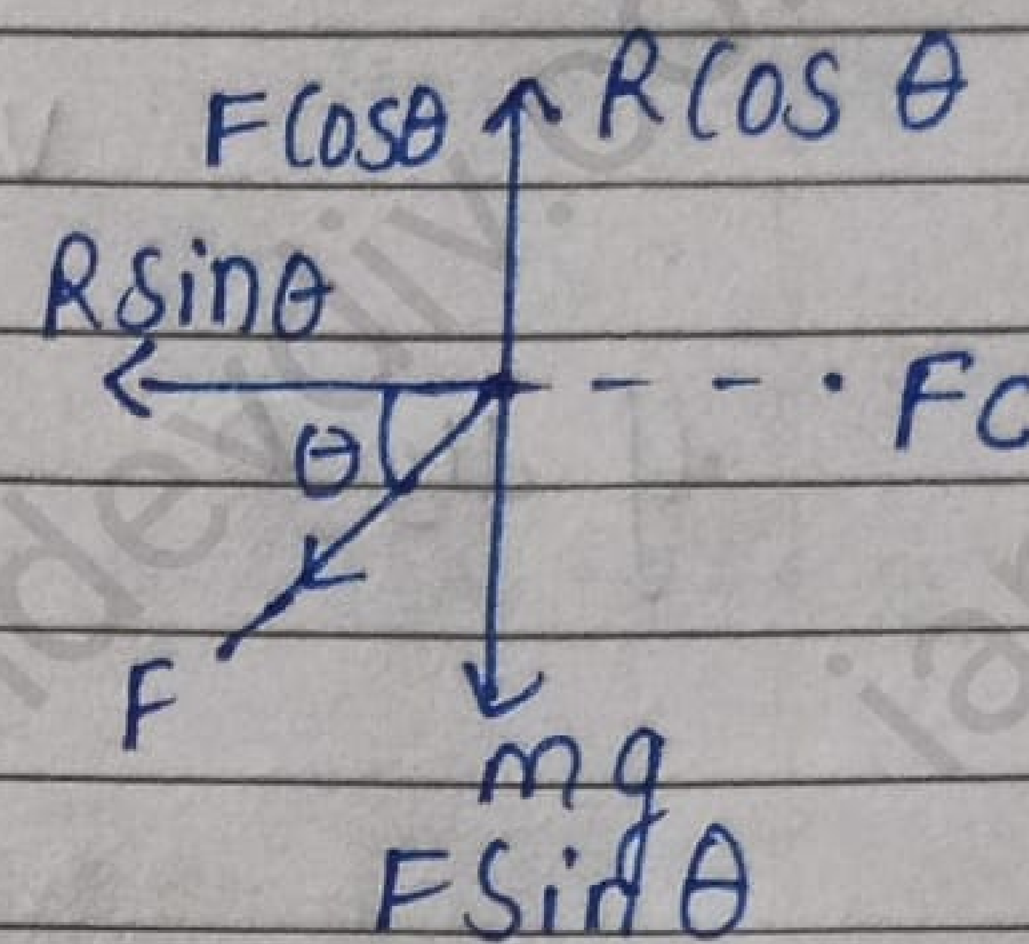
$v^2 \leq urg$

$\boxed{v \leq \sqrt{urg}}$

$\boxed{v_{max} = \sqrt{urg}}$

☆ Banking of Road





$$R\cos\theta = mg + F\sin\theta$$
$$R\cos\theta - F\sin\theta = mg$$
$$R\cos\theta - \mu_s R\sin\theta = mg$$
$$R(\cos\theta - \mu_s \sin\theta) = mg$$
$$R = \frac{mg}{(\cos\theta - \mu_s \sin\theta)} \quad - (1)$$

Now again

$$F_c = F\cos\theta + R\sin\theta$$
$$\frac{mv^2}{r} = u_s R\cos\theta + R\sin\theta$$
$$\frac{mv^2}{r} = R(u_s \cos\theta + \sin\theta) \quad - (2)$$

from (1) & (2)

$$\frac{\cancel{m}v^2}{r} = \frac{\cancel{m}g}{(\cos\theta + u_s \sin\theta)}(\sin\theta + u\cos\theta)$$
$$v^2 = rg\left[\frac{\sin\theta + u_s\cos\theta}{\cos\theta - u_s\sin\theta}\right]$$
$$= rg\left[\frac{\frac{\sin\theta + u_s\cos\theta}{\cos\theta}}{\frac{\cos\theta - u_s\sin\theta}{\cos\theta}}\right]$$

$$v^2 = rg\left[\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right]$$

$$v = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)} \quad - (3)$$

if $\mu_s = 0$

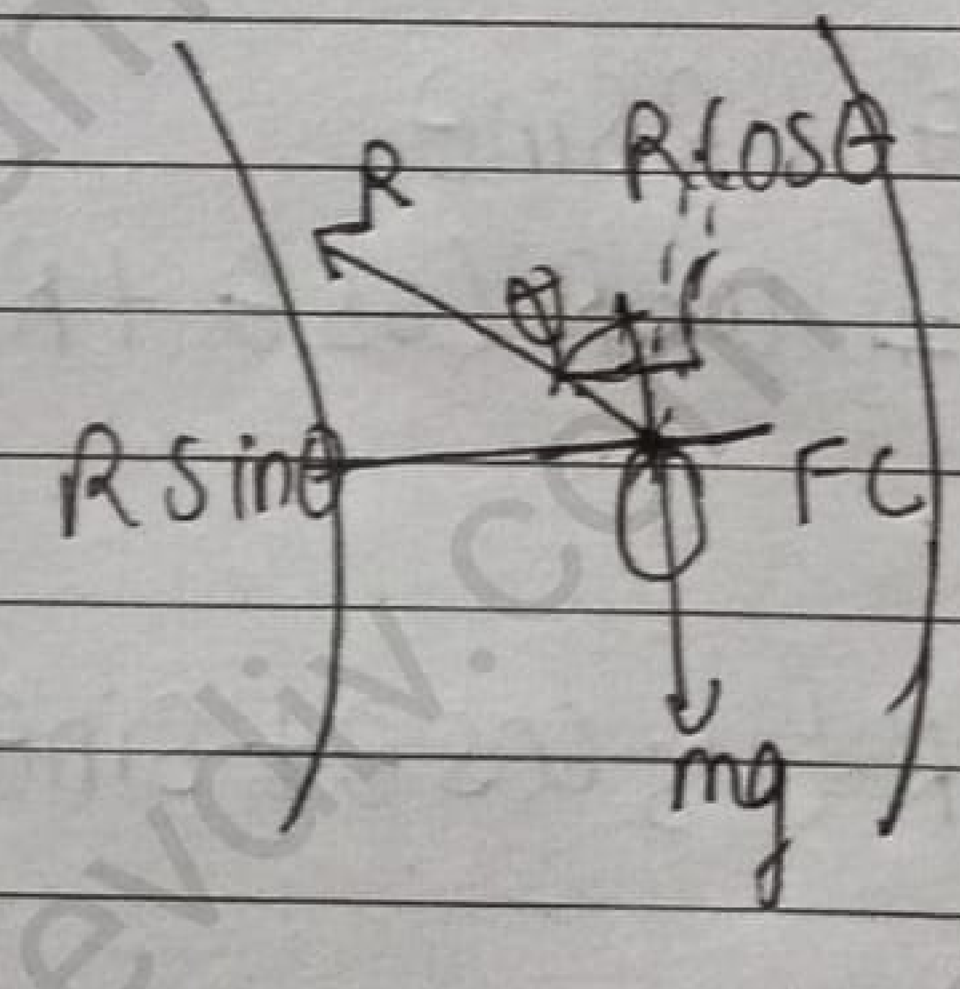
$$v = \sqrt{rg\left[\frac{\tan\theta + 0}{1 - 0}\right]}$$

$$v = \sqrt{rg\tan\theta}$$

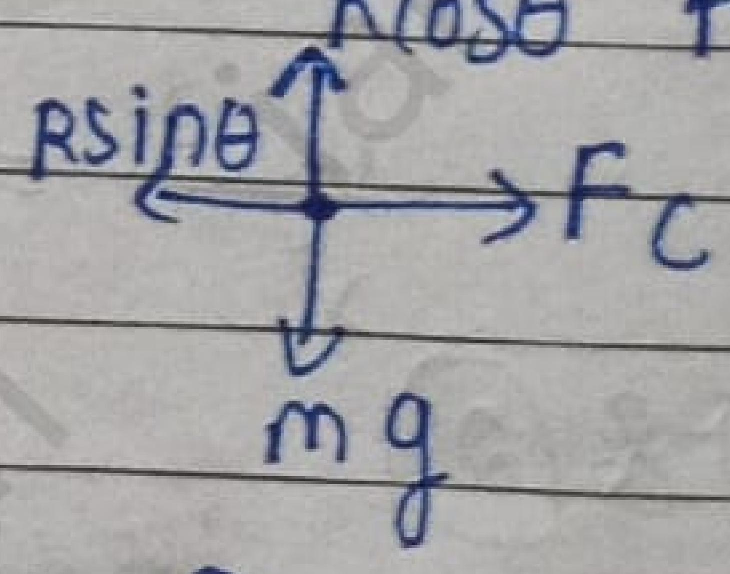
$$v = \sqrt{rg\tan\theta} = v^2 = rg\tan\theta$$

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

#



Avoid friction force



$$R\sin\theta = \frac{mv^2}{r} \quad - (1)$$

$$R\cos\theta = mg \quad - (2)$$

(1)/(2)

$$\frac{\cancel{R}\sin\theta}{\cancel{R}\cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g} = \tan\theta = \frac{v^2}{rg}$$